Final Exam (Version 1) - Math 141, Frank Thorne (thorne@math.sc.edu)

Wednesday, December 13, 2023

Please work without books, notes, calculators, phones, or assistance from others. If you have any questions, ask. Please do your work on separate paper and turn that in.

GOOD LUCK!

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) What is the *definite integral* of a function $f(x)$, from $x = a$ to $x = b$? (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the signed area under the graph of $f(x)$ between $x = a$ and $x = b$.

(3) What does the Fundamental Theorem of Calculus say? (Both parts) Why is it important?

(4) Find the derivative of

$$z = \frac{4 - 3x}{3x^2 + x}.$$

(5) Find the derivative of

$$q = \sqrt[3]{2r - r^2}.$$

(6) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

(7) Graph the function $f(x) = \frac{4x}{x^2+4}$.

Answer the following questions as part of your solution:

(a) Where are the critical points of f?

(b) Where the local and absolute maxima and minima of f?

(c) Where are the inflection points of f?

(d) Where is f increasing and decreasing?

(e) Where is f concave up and down?

(8) Find the point on the line $\frac{x}{a} + \frac{y}{b} = 1$ that is closest to the origin.

(9) Evaluate

$$\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4)dt.$$

(10) Evaluate

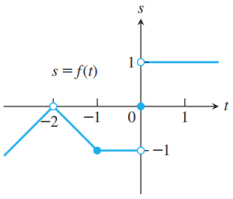
$$\int \frac{1}{\sqrt{5s+4}}\, ds.$$

(11) Find the area of a square pyramid with base length b and height b.

(12) (See below)

2. For the function $f(t)$ graphed here, find the following limits or explain why they do not exist.

a. $\lim_{t\to -2} f(t)$ **b.** $\lim_{t\to -1} f(t)$ **c.** $\lim_{t\to 0} f(t)$ **d.** $\lim_{t\to -0.5} f(t)$



(13) Differentiate the function

$$y = f(x) = \frac{8}{\sqrt{x-2}}$$

using the definition of the derivative directly. Then find an equation of the tangent line at the point $(x, y) = (6, 4)$ on the graph of the function.

No credit for using the power/quotient rules etc. for the first part, but use them if needed to answer the second part.

(14) Compute dy/dx if

$$y = \arccos(x^2).$$

(15) Find

$$\lim_{h\to 0} \frac{e^h - (1+h)}{h^2}.$$

(16) Using rectangles, each of whose height is given by the value of the function at the midpoint of the rectangle's base (the midpoint rule), estimate the area under the graph of the function

$$f(x) = x^2$$

from $x = 0$ to $x = 1$, using two rectangles.

Be sure to draw a picture and simplify your answer.