Final Exam (Version 2) - Math 141, Frank Thorne (thorne@math.sc.edu)

Wednesday, December 13, 2023

Please work without books, notes, calculators, phones, or assistance from others. If you have any questions, ask. Please do your work on separate paper and turn that in.

GOOD LUCK!

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) What is the *definite integral* of a function $f(x)$, from $x = a$ to $x = b$? (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the signed area under the graph of $f(x)$ between $x = a$ and $x = b$.

(3) What does the Fundamental Theorem of Calculus say? (Both parts) Why is it important?

(4) Find the derivative of

$$g(x) = \frac{x^2 - 4}{x + 0.5}.$$

(5) Find dy/dt if

$$y = \frac{1}{6}(1 + \cos^2(7t))^3.$$

(6) Water is flowing at the rate of 50 m^3 per minute from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

How fast (in centimeters per minute) is the water level falling when the water is 5 m deep?

(7) Graph the function $f(x) = \cos x + \sqrt{3}\sin x$, $0 \leq x \leq 2\pi$.

Answer the following questions as part of your solution:

(a) Where are the critical points of f?

(b) Where the local and absolute maxima and minima of f?

(c) Where are the inflection points of f?

(d) Where is f increasing and decreasing?

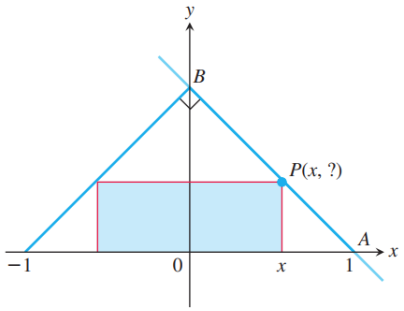
(e) Where is f concave up and down?

(8) The figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.

(a) Express the y-coordinate of P in terms of x. (Hint: Write an equation for the line AB.)

(b) Express the area of the rectangle in terms of x.

(c) What is the largest area the rectangle can have, and what are its dimensions?



(9) Evaluate

$$\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} ds.$$

(10) Evaluate

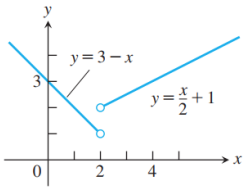
$$\int \frac{e^{\arccos x}}{\sqrt{1-x^2}} dx.$$

(11) Find the volume of the solid generated by generating the region bounded by these lines and curves around the x-axis.

$$y = x^3, \quad y = 0, \quad x = 2.$$

(12) (See below)

3. Let $f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & x > 2. \end{cases}$



a. Find $\lim_{x \to 2^+} f(x)$ and $\lim_{x \to 2^-} f(x)$.

b. Does $\lim_{x \to 2} f(x)$ exist? If so, what is it? If not, why not?

c. Find $\lim_{x \to 4^-} f(x)$ and $\lim_{x \to 4^+} f(x)$.

d. Does $\lim_{x \to 4} f(x)$ exist? If so, what is it? If not, why not?

(13) Compute the derivative of $r(s) = \sqrt{2s+1}$ using the definition (and not the power/chain/etc. rules), and compute $r'(0)$.

(14) Use implicit differentiation to find dy/dx and then d^2y/dx^2, if

$$x^{2/3} + y^{2/3} = 1.$$

Write the solutions in terms of x and y only.

(15) Solve the initial value problem

$$\frac{d^2y}{dx^2} = 2 - 6x, \quad y'(0) = 2, \quad y(0) = 0.$$

(16) Find the total area between the region

$$y = -x^2 - 2x, \quad -3 \leq x \leq 2$$

and the x-axis. (Do not compute the signed area: instead, count all area as positive.)