Final Exam (Version 4)) - Math 141, Frank Thorne (thorne@math.sc.edu)

Wednesday, December 13, 2023

Please work without books, notes, calculators, phones, or assistance from others. If you have any questions, ask. Please do your work on separate paper and turn that in.

GOOD LUCK!

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) What is the *definite integral* of a function $f(x)$, from $x = a$ to $x = b$? (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the signed area under the graph of $f(x)$ between $x = a$ and $x = b$.

(3) What does the Fundamental Theorem of Calculus say? (Both parts) Why is it important?

(4) Find ds/dt, if
$$s = \frac{1 + \csc t}{1 - \csc t}.$$

(5) Find dy/dt if
$$y = \sec^2 \pi t.$$

(6) Sand falls from a conveyor belt at the rate of 10 m^3 per minute onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high? Answer in centimeters per minute.

(7) Graph the function $f(x) = \sqrt{|x - 4|}$.

Answer the following questions as part of your solution:

(a) Where are the critical points of f?

(b) Where the local and absolute maxima and minima of f?

(c) Where are the inflection points of f?

(d) Where is f increasing and decreasing?

(e) Where is f concave up and down?

(8) Find a positive number for which the sum of it and its reciprocal is the smallest (least) possible.
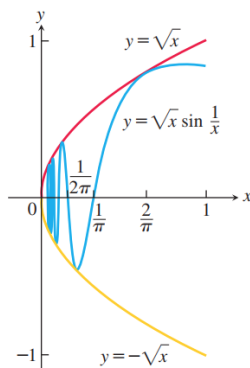
(9) Evaluate

$$\int_0^{\pi/8} \sin 2x \ dx.$$

(10) Evaluate

$$\int \sqrt{3-2s} \ dx.$$

(11) Find the volume of a circular cone of radius r and height h.

(12) (see below)

6. Let $g(x) = \sqrt{x}\sin(1/x)$.

a. Does $\lim\limits_{x\to 0^+} g(x)$ exist? If so, what is it? If not, why not?

b. Does $\lim\limits_{x\to 0^-} g(x)$ exist? If so, what is it? If not, why not?

c. Does $\lim\limits_{x\to 0} g(x)$ exist? If so, what is it? If not, why not?

(13) Compute

$$\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$$

and

$$\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}.$$

(14) If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0,-1)$.

(15) Evaluate

$$\int \left(\frac{1}{7} - \frac{1}{y^{5/4}}\right) dy.$$

(16) (see below)

13.

y

$\sqrt{2}$

$y = \sec\theta\tan\theta$

$-\dfrac{\pi}{4}$

0

$\dfrac{\pi}{4}$

θ

$-\sqrt{2}$