Final Exam (Version 5) - Math 141, Frank Thorne (thorne@math.sc.edu)

Wednesday, December 13, 2023

Please work without books, notes, calculators, phones, or assistance from others. If you have any questions, ask. Please do your work on separate paper and turn that in.

GOOD LUCK!

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) What is the *definite integral* of a function $f(x)$, from $x = a$ to $x = b$? (Please give the algebraic definition, using an equation.)

Draw a picture and explain why your equation gives the signed area under the graph of $f(x)$ between $x = a$ and $x = b$.

(3) What does the Fundamental Theorem of Calculus say? (Both parts) Why is it important?

(4) Find the derivative of

$$u = \frac{5x+1}{2\sqrt{x}}.$$

(5) Find the derivative of

$$y = \frac{1}{18}(3x-2)^6 + \left(4 - \frac{1}{2x^2}\right)^{-1}.$$

(6) The coordinates of a particle in the metric xy-plane are differentiable functions of time t with $dx/dt = -1\ m/sec$ and $dy/dt = -5\ m/sec$. How fast is the particle's distance from the origin changing as it passes through the point $(5, 12)$?

(7) Graph the function $f(x) = 2x - 3x^{2/3}$.

Answer the following questions as part of your solution:

(a) Where are the critical points of f?

(b) Where the local and absolute maxima and minima of f?

(c) Where are the inflection points of f?

(d) Where is f increasing and decreasing?

(e) Where is f concave up and down?

(8) You are designing a 1000 cm^3 right circular cylindrical can whose manufacture will take waste into account. There is no waste in cutting the aluminum for the side, but the top and bottom of radius r will be cut from squares that measure $2r$ units on a side. The total amount of aluminum used up by the can will therefore be

$$A = 8r^2 + 2\pi rh,$$

rather than $A = 2\pi r^2 + 2\pi rh$ as in Example 2 in the book (where waste was not taken into account).

In Example 2, the ratio of h to r for the most economical can was 2 to 1. What is the ratio for the most economical can now?

(9) Evaluate

$$\int_0^{\pi/6} (\sec x + \tan x)^2 \, dx.$$

(10) Solve the initial value problem

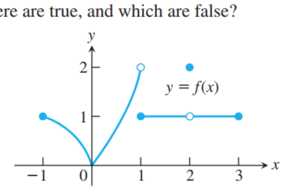
$$\frac{ds}{dt} = 12t(3t^2 - 1)^3,$$

with $s(1) = 3$.

(11) Find the volume of a sphere with radius r.

(12) (See below)

2. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



a. $\lim_{x \to -1^+} f(x) = 1$ **b.** $\lim_{x \to 2} f(x)$ does not exist.

c. $\lim_{x \to 2} f(x) = 2$ **d.** $\lim_{x \to 1^-} f(x) = 2$

e. $\lim_{x \to 1^+} f(x) = 1$ **f.** $\lim_{x \to 1} f(x)$ does not exist.

g. $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(13) Compute

$$\lim_{x \to 0^+} \frac{|\sin x|}{x}$$

and

$$\lim_{x \to 0^-} \frac{|\sin x|}{x}.$$

(14) Find dy/dt if

$$y = \operatorname{arccot} \sqrt{t}.$$

(15) Evaluate

$$\int (1.3)^x \, dx.$$

(16) Compute

$$\int_0^1 \frac{x^3}{\sqrt{x^4+9}} \, dx.$$