Study Guide AP Calc – Integrals

Unit 6 – Concepts Involving Integration

- Area Approximation Strategies
  - Rectangular Approximation Methods (LRAM, RRAM, MRAM)
    - $area = \sum_{i=1}^{n} f(xi) * \Delta x$
    - n = number of rectangles, f(xi) = length of rectangles, i = width of rectangles, $\Delta x = \frac{b-a}{n}$
  - Trapezoidal Rule for Approximation
    - $area = \frac{\Delta x}{2}[f(x1) + 2f(x2) + \cdots + 2f(xn) + f(xn+1)]$
- Properties of Integrals
  - Addition Property – if a < b < c then,
    - $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
  - Coefficient Property – for any real number c
    - $\int_a^b c\, f(x)dx = c \int_a^b f(x)dx$
  - Bounds Property
    - $\int_a^b f(x)dx = -\int_b^a f(x)dx$
  - Integral Sum and Difference Property
    - $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
- Limit Definition of The Integral
  - $\lim_{n\to\infty} \sum_{i=1}^{n} f(xi)\Delta x = \int_a^b f(x)dx$
  - $\Delta x = \frac{b-a}{n} \quad xi = a + i * \Delta x$
    - $\lim_{n\to\infty} \sum_{i=1}^{n} (4\left(\frac{5i}{n}\right)) = \int_0^5 4x dx$
- Definition for Power Rule of Integration
  - Inverse of Derivative, denoted as F(x)
  - $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
    - Increase Power of x term by one
    - Divide by new power
    - Add constant, C
    - Simplify
  - n must be rational and not equal to -1, C is constant of Integration
- Fundamental Theorem of Calculus
  - Definite Integrals refer to an integral that has an upper and lower limit which can be evaluated to a specific value
  - If f is continuous on [a,b], then
    - $\int_a^b f(x)dx = [F(b) + C] - [F(a) + C] = F(b) - F(a)$
    - Where F is any antiderivative of f, that is, a function such that F'(x) = f(x)
      - Find antiderivative (Integrate)
      - Evaluate F(b) – F(a), ignore constants
      - Simplify
  - If f is a continuous function on [a,b] and x is a point in (a,b), then
    - $\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$
    - or if $g(x) = \int_a^x f(t)dt$, then g'(x) = f(x)
- Summary of FTOC Connections when Given the Graph of $f(x) = \int_a^x g(t)dt$
  - f'(x) = g(x)
  - Increasing above axis, Decreasing below x axis
  - Concave up where graph is increasing, Concave down where graph is decreasing
  - Local Max where graph switches above to below, Local Min where graph switches below to above
  - Absolute Max and Min using EVT
  - Point of Inflection where graph switches from increasing to decreasing

Unit 7 – Rules/Techniques of Integration

- Summary of U-Substitution Method
  - Use U-Sub when a quantity is raised to a power, trig ratio raised to a power, and trig ratios of unusual angles
  - $\int f(g(x))dx = \int f(u)du$
    - U equals inside quantity, trig ratio raised to a power, unusual angle, or both trig and unusual angle
    - Find $\frac{du}{dx}$
    - Match $\frac{du}{dx}$ to original function
    - Substitute
    - Find antiderivative
    - Re-substitute for x
- List of Trigonometric Integrals

o	f(x) = sin(x)	o	F(x) = -cos(x) + c
o	f(x) = cos(x)	o	F(x) = sin(x) + c
o	f(x) = $\sec^2$(x)	o	F(x) = tan(x) + c
o	f(x) = csc(x) cot(x)	o	F(x) = -csc(x) + c
o	f(x) = sec(x) tan(x)	o	F(x) = sec(x) + c
o	f(x) = $\csc^2$(x)	o	F(x) = -cot(x) + c

- List of Inverse Trigonometric Integrals

o	f(x) = $\frac{1}{\sqrt{1-x^2}}$	o	F(x) = $\sin^{-1}$(x) + c

○ f(x) = $\frac{1}{1+x^2}$	○ F(x) = $\tan^{-1}(x) + c$

- List of Logarithmic and Exponential Integrals – Natural Base and Other Bases

○ f(x) = $\frac{1}{x}$	○ F(x) = $\ln\|x\| + c$
○ f(x) = e^x	○ F(x) = $e^x + c$
○ f(x) = a^x	○ F(x) = $\frac{a^x}{\ln a} + c$

- Separating Differential Equations
  - Differential Equation – Equation containing derivative of function or equation
  - Finding Integrals for Implicit Derivatives
    - Use differentials to get like variables on same side
    - Find antiderivative of each side
    - Put into y = form if possible
- Slope Fields
  - Slope fields – tool that can be used to approximate shape of graph given the derivative
    - Choose points in the field around initial value if one is given
    - Plug into derivative equation to determine slope at point
    - Sketch slope on that point in the graph
    - Draw a curve following slopes and initial value

Unit 8 – Applications of Integration

- Total Change Theorem
  - $\int_a^b F'(x)dx = F(b) - F(a)$
  - Calculates total change in a scenario
- Average Value for Integrals
  - If f is continuous on a closed interval [a,b] and differentiable on the open interval (a,b), then the average value of f from x = a to x = b is given by
    - $Average\ Value = \frac{1}{b-a}\int_a^b f(x)dx$
- Mean Value Theorem for Integrals
  - If f is continuous on a closed interval [a,b] and differentiable on the open interval (a,b), then there exists at least one value c where
    - $\frac{1}{b-a}\int_a^b f(x)dx = f(c)$
- Calculating Displacement vs. Total Distance Traveled from v(t)
  - Displacement
    - $\int_a^b v(t)dt$
    - Difference between ending and starting points
  - Distance
    - $\int_a^b |v(t)|dt$
    - Total distance traveled
- Calculating Area of an Enclosed Region
  - Area between a curve f(x) and the x or y axis is defined to be $\int_a^b f(x)dx$ or $\int_a^b f(y)dy$ above or to the right of the axes and $-\int_a^b f(x)dx$ or $-\int_a^b f(y)dy$ below or to the left of the axes on the interval [a,b]
  - Area between two curves f(x) and g(x) or f(y) and g(y) in [a,b] is defined to be $\int_a^b [f(x) - g(x)]dx$ or $\int_a^b [f(y) - g(y)]dy$ if f is greater than g
- Disk Method Formula with Respect to both x and y Axis
  - X axis, Y=n
    - $V = \int_a^b \pi[f(x)]^2 dx$
  - Y Axis, X=n
    - $V = \int_a^b \pi[f(y)]^2 dy$
- Washer Method Formula with Respect to both x and y Axis
  - X Axis, Y=n
    - $V = \int_a^b \pi[f(x)^2 - g(x)^2]dx$
  - Y Axis, X=n
    - $V = \int_a^b \pi[f(y)^2 - g(y)^2]dy$
- Shell Method Formula with Respect to both x and y Axis
  - X Axis, Y=n
    - $V = \int_{y=a}^{y=b} 2\pi y[f(y)]dy$
    - Right - Left
  - Y Axis, X=n
    - $V = \int_{x=a}^{x=b} 2\pi x[f(x)]dx$
    - Upper - Lower
- Volume for Solid with Known Cross Section (Slabs)
  - Perpendicular to X Axis
    - $V = \int_{x=a}^{x=b} A(x)dx$
  - Perpendicular to Y Axis
    - $V = \int_{y=a}^{y=b} A(y)dy$
    - A is area of the cross section
      - Square – s^2
      - Equilateral Triangle - $\frac{\sqrt{3}}{4}b^2$
      - Semi-Circle - $\frac{\pi r^2}{2}$
      - Isosceles Right Triangle - $\frac{b^2}{2}$