1. The function $f(x) = 5 - 12x + 3x^2$ is continuous and differentiable everywhere, because it is a polynomial.

Also, $f(1) = 5 - 12 + 3 = -4$

$f(3) = 5 - 12 \cdot 3 + 3 \cdot 9 = -4 = f(1)$.

So $f(x)$ satisfies Rolle's Theorem on $[1, 3]$:

$f'(x) = 0$ for some c with $1 < c < 3$.

$f'(x) = -12 + 6x$.

If $f'(x) = -12 + 6x = 0$ then $6x = 12$

$x = 2$.

So we take $c = 2$.

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2. $h(x) = x^5 - 2x^3 + x$

$h'(x) = 5x^4 - 6x^2 + 1$

$= (5x^2 - 1)(x^2 - 1)$

$h''(x) = 20x^3 - 12x = 4x(5x^2 - 3)$.

If $h'(x) = 0$ then $5x^2 - 1 = 0$ or $x^2 - 1 = 0$

so $5x^2 = 1$ or $x^2 = 1$

so $x = \pm \frac{1}{\sqrt{5}}$ or $x = \pm 1$.

2. $h(x) = x^5 - x$
$h'(x) = 5x^4 - 1$
$h''(x) = 20x^3$.

If $h'(x) = 0$ then $x^4 = \frac{1}{5}$, so $x = \frac{\pm 1}{\sqrt[4]{5}}$.

Sign of $h'(x)$: $\left(-\infty, \frac{-1}{\sqrt[4]{5}}\right)$ $\left(\frac{-1}{\sqrt[4]{5}}, \frac{1}{\sqrt[4]{5}}\right)$ $\left(\frac{1}{\sqrt[4]{5}}, 1\right)$

$+$ $-$ $+$

To see this: $h'(-1) = 4$
$h'(1) = 4$
$h'(0) = -1$.

Sign of $h''(x)$ is positive if $x > 0$
negative if $x < 0$.

So: $h(x)$ has critical points at $x = \frac{\pm 1}{\sqrt[4]{5}}$.

$\frac{1}{\sqrt[4]{5}}$ is a minimum because h is concave up

$\frac{-1}{\sqrt[4]{5}}$ is a maximum because h is concave down

$h(x)$ is concave up for $x > 0$ $(h''(x) > 0)$
concave down for $x < 0$ $(h''(x) < 0)$.

$h(x)$ is increasing on $\left(-\infty, \frac{-1}{\sqrt[4]{5}}\right)$ and $\left(\frac{1}{\sqrt[4]{5}}, 1\right)$
decreasing on $\left(\frac{-1}{\sqrt[4]{5}}, \frac{1}{\sqrt[4]{5}}\right)$.

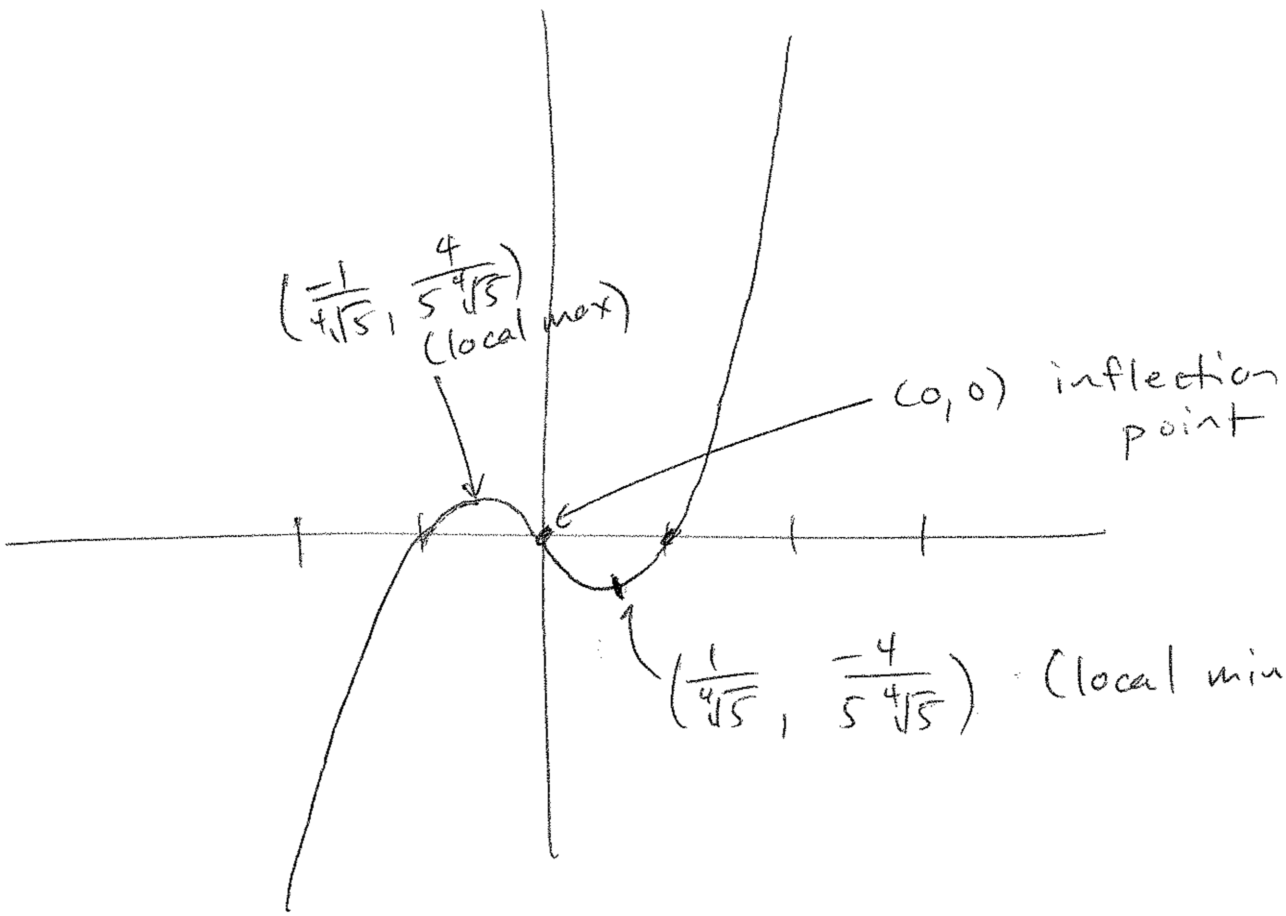
If $x = 0$, $h(x) = 0$.

If $x = \frac{1}{\sqrt[4]{5}}$, $h(x) = \frac{1}{5 \cdot \sqrt[4]{5}} - \frac{1}{\sqrt[4]{5}} = \frac{-4}{5 \cdot \sqrt[4]{5}}$.

If $x = \frac{-1}{\sqrt[4]{5}}$, $h(x) = \frac{+4}{5 \cdot \sqrt[4]{5}}$.

If $x = 1$, $h(x) = 0$,

If $x = 2$, $h(x) = 30$.

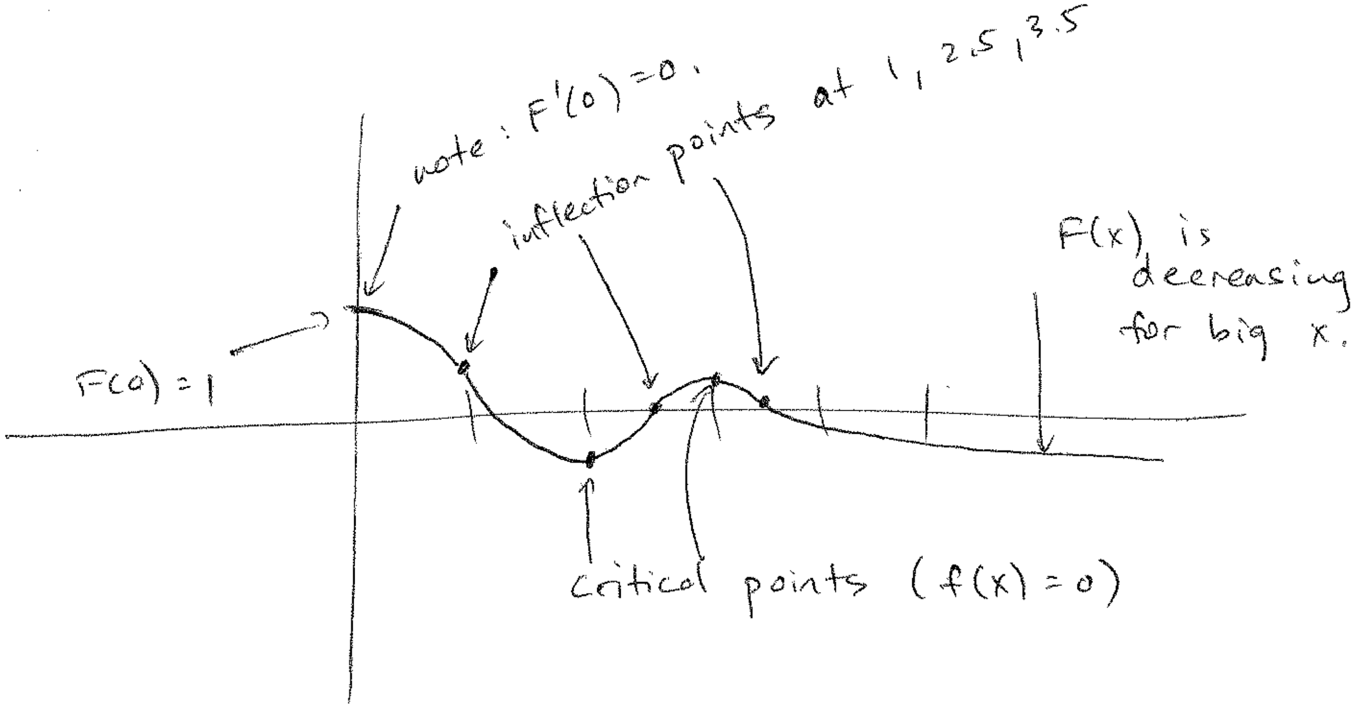


3. F is decreasing on $(0,2)$ and $(3,\infty)$

increasing on $(2,3)$

F is Concave up when $f'(x) > 0$: $(1, 2.5)$ and $(3.5, \infty)$

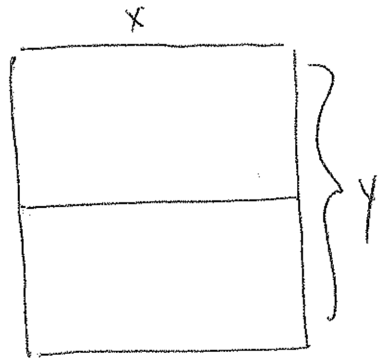
Concave down when $f'(x) < 0$: $(0,1)$ and $(2.5, 3.5)$



F has critical points when $f(x) = 0$, so $x = 0, 2, 3$.

possible inflection points when the graph of $f(x)$ is flat, so $x = 1$ and (about) $2.5, 3.5$.

9.



Let x = width of fence

y = length

Then area $= 15000 \text{ ft}^2 = x \cdot y$.

Let F = total length of fence

$= 3x + 2y$.

Since $x = \dfrac{15000}{y}$, we have

$$F(y) = 3 \cdot \frac{15000}{y} + 2y$$

$$= \frac{45000}{y} + 2y.$$

If $\dfrac{dF}{dy} = 0$, $\quad -\dfrac{45000}{y^2} + 2 = 0$

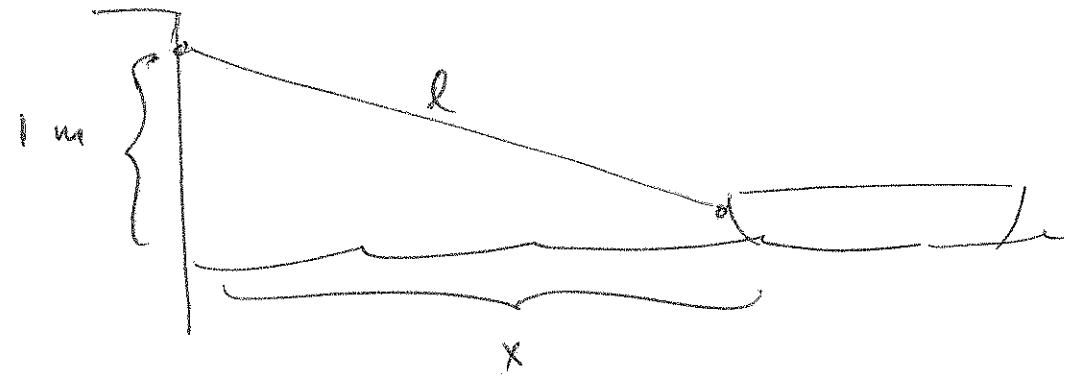
$$2 = \frac{45000}{y^2}$$

$y^2 = 22,500$ so $y = 150$.

This means $x = \dfrac{15000}{150} = 100$.

The fence should be 100 ft × 150 ft.

5.



Let ℓ = length of rope
x = distance from dock to boat

Know: $1 + \ell^2 = x^2$ and $\frac{d\ell}{dt} = -1$ m/s.

So $2\ell \frac{d\ell}{dt} = 2x \frac{dx}{dt}$, so $2x \frac{dx}{dt} = -2\ell$.

So $\frac{dx}{dt} = \frac{-2\ell}{2x}$.

When $x = 8$ m, $\ell = \sqrt{8^2 + 1^2} = \sqrt{65}$.

So $\frac{dx}{dt} = \frac{-\ell}{x} = -\frac{\sqrt{65}}{8}$

The boat is approaching the dock at $\frac{\sqrt{65}}{8}$ m/s.

(This is slightly bigger than 1.)

6. $\int_{-2}^{1} x^{-4}\,dx$ is not defined because 0^{-4} is not defined.

$\int_a^b f(x)\,dx$ only makes sense if $f(x)$ is defined between a and b.

7. $$\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 \left(4u^{-3} + u^{-1}\right) du$$

$$= 4\cdot\frac{u^{-2}}{-2} + \ln|u| \Big]_1^2$$

$$= \left(\frac{4/4}{-2} + \ln(2)\right) - \left(\frac{4\cdot 1}{-2} + \ln(1)\right)$$

$$= \frac{-1}{2} + \ln 2 + 2 = \frac{3}{2} + \ln 2.$$