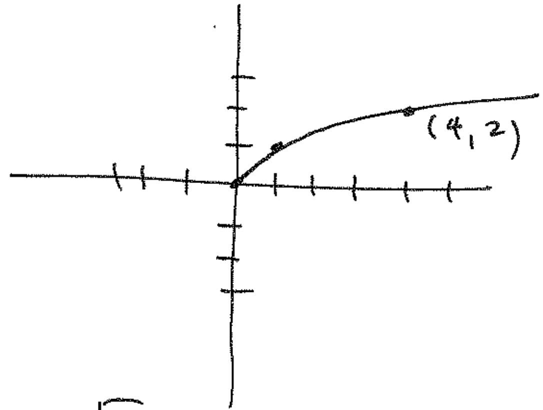
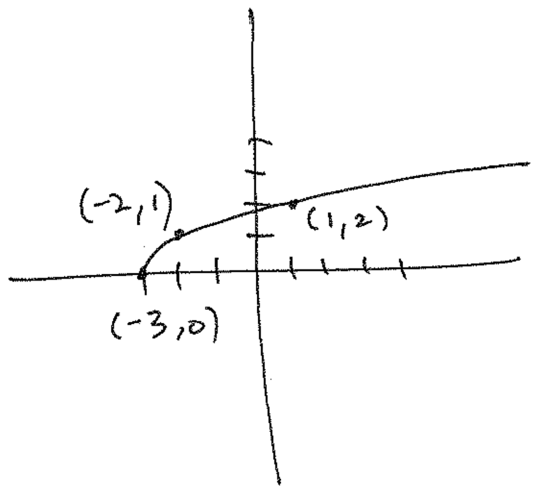
1. $y = \sqrt{x}$ looks like this



If (a,b) is a point on $y = \sqrt{x}$, then $(a-3, b)$ is a point on $y = \sqrt{x+3}$ because $b = \sqrt{a}$ is the same as $b = \sqrt{(a-3)+3}$. So we shift the graph by 3 to the left:



2. If $f(x)$ is an ~~reverse~~ function. and $f(x) = y$, then the inverse function is the function that undoes f. In other words $f^{-1}(y) = x$ whenever $f(x) = y$.

The function $f(x) = x + 5$ has an inverse because we can always undo it. This is because $f(x)$ is <u>one-to-one</u>. It never takes the same value twice.

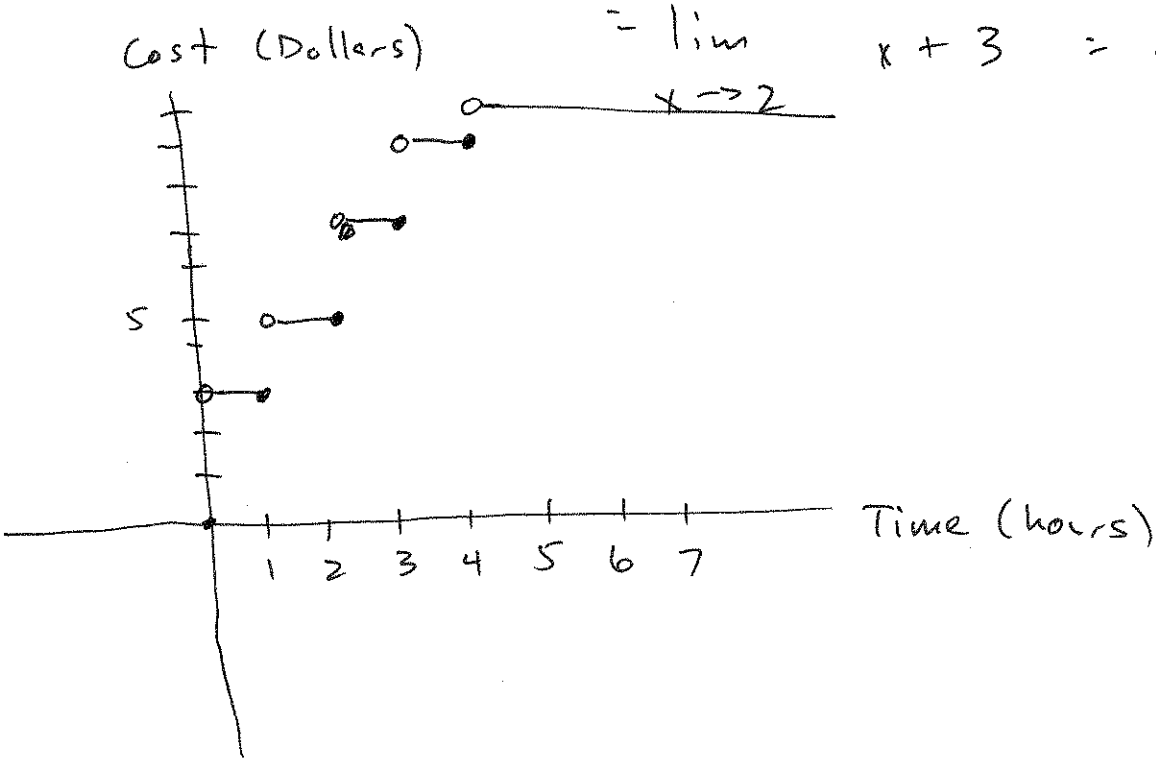
The function $f(x) = x^2$ does not have an inverse because it is not one-to-one. For example, $f(-2) = f(2) = 4$, so should $f^{-1}(4)$ be 2 or -2? There is no way to decide.

3.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+3)}{x-2}$$

$$= \lim_{x \to 2} x + 3 = 2 + 3 = 5.$$

4.



The function is discontinuous at $x = 0, 1, 2, 3, 4$ because it jumps. This means the cost of parking jumps suddenly. If you get to your car just before it jumps, then you can save a noticeable amount of money.

5. Done in class.

6. (a) $f'(x)$ is in dollars per ounce. It is the cost per ounce of producing more gold, after you have already produced x ounces.

(b) $f'(800) = 17$ means that once 800 ounces have been mined, the cost of producing a small amount more of gold is 17 dollars/oz.

(c) I expect $f'(x)$ to decrease in the short term. The first little bit of gold might be hard to mine because the miners don't know where the most gold is yet, or don't have their equipment set up optimally, but as the mining operation proceeds this will become more efficient.

In the long term $f'(x)$ will increase, because the miners will find all the easily accessible gold and mine it, and later the miners will have to mine in more difficult or less concentrated spots.

7. $\lim\limits_{x \to \infty} \sqrt{9x^2 + x} - 3x = \lim\limits_{x \to \infty} \dfrac{(\sqrt{9x^2+x} - 3x)(\sqrt{9x^2+x} + 3x)}{\sqrt{9x^2+x} + 3x}$

$$= \lim_{x \to \infty} \frac{9x^2 + x - (3x)^2}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x \to \infty} \frac{x}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x \to \infty} \frac{\frac{x}{x}}{\sqrt{\frac{9x^2}{x^2} + \frac{x}{x^2}} + \frac{3x}{x}}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3}$$

$$= \frac{1}{\sqrt{9} + 3} \quad \left(\text{because } \frac{1}{x} \to 0 \text{ as } x \to \infty\right)$$

$$= \frac{1}{3+3} = \frac{1}{6}.$$

8. $$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{(3 - 2(a+h) + 4(a+h)^2) - (3 - 2a + 4a^2)}{h}$$

$$= \lim_{h\to 0} \frac{3 - 2a - 2h + 4a^2 + 8ah + 4h^2 - 3 + 2a - 4a^2}{h}$$

$$= \lim_{h\to 0} \frac{-2h + 8ah + 4h^2}{h}$$

$$= \lim_{h\to 0} -2 + 8a + 4h = -2 + 8a.$$