Quiz 1 - Math 544, Frank Thorne (thorne@math.sc.edu)

Monday, August 31, 2015

Describe the set of all vectors in $\mathbb{R}^2$ which are orthogonal to $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$. Draw the relevant picture and explain it thoroughly.

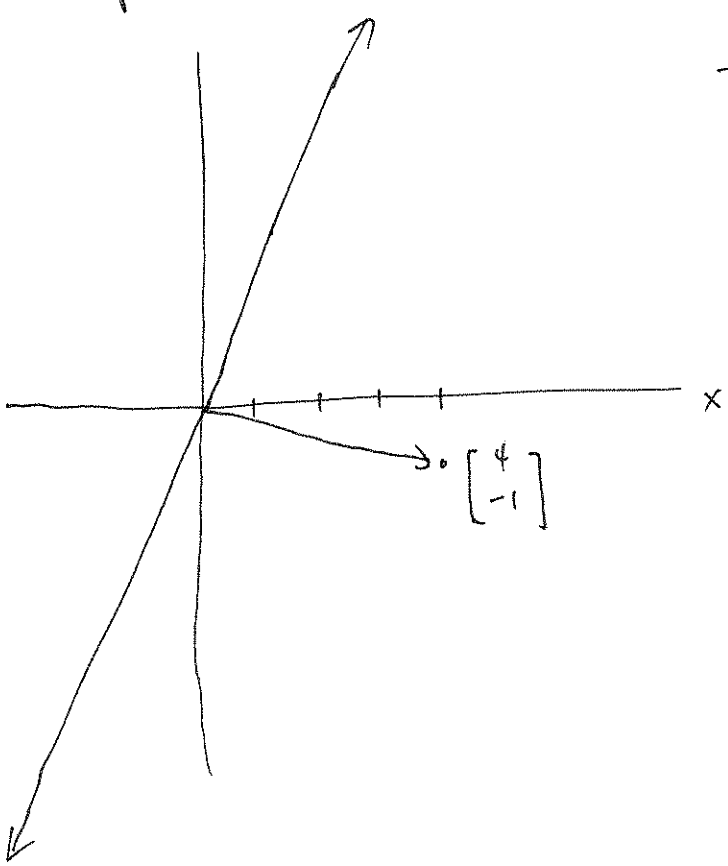
This is the set $\left\{ \vec{v} \in \mathbb{R}^2 : \vec{v} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 0 \right\}$

$$= \left\{ \vec{v} \in \mathbb{R}^2 : \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 0 \right\}$$

$$= \left\{ \vec{v} \in \mathbb{R}^2 : 4x - y = 0 \right\}$$

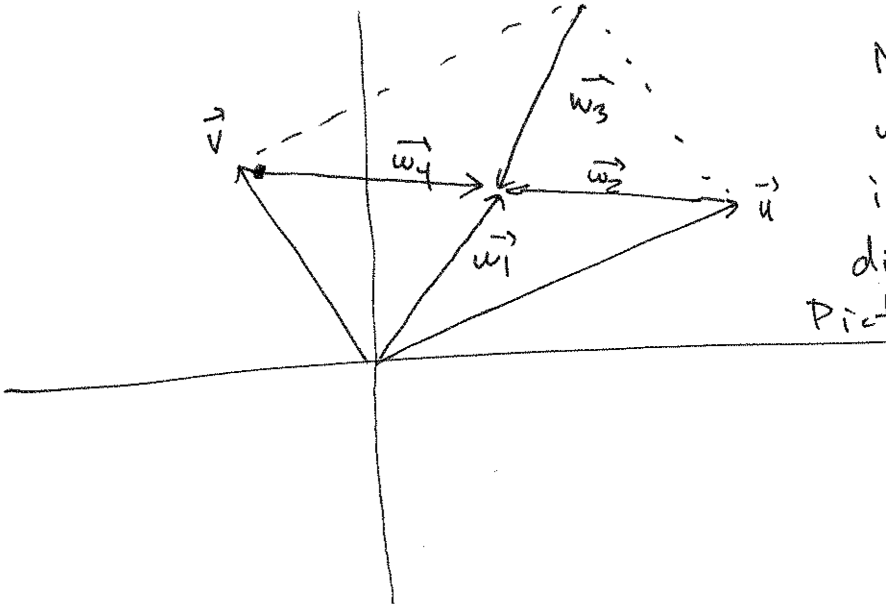
i.e. the set of solutions ~~to~~ to the equation $4x - y = 0$, which forms a line.

If we let $\vec{v}_\ell$ be any nonzero vector in this set, e.g. $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$, then we can also describe the line as the set of all scalar multiples of $\vec{v}_\ell$.



The solution set is the line $y = 4x$, and it is also the set of vectors which make right angles to $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

1.1, B4.



Note: $\vec{w_1}$ and $\vec{w_3}$ should be in exactly opposite directions. Picture is drawn a bit poorly.

(a) $\vec{w_1} = \frac{1}{2}(\vec{u} + \vec{v})$, because $\vec{u}+\vec{v}$ ends at the opposite vertex (from the origin) and $\vec{w_1}$ is halfway there.

$\vec{w_2} = \frac{1}{2}(\vec{v} - \vec{u})$, because it is in the direction from $\vec{u}$ to $\vec{v}$, and half the distance.

(b) $\vec{w_3} = -\vec{w_1} = -\frac{1}{2}(\vec{u} + \vec{v})$

$\vec{w_4} = -\vec{w_2} = \frac{1}{2}(\vec{u} - \vec{v})$.

By arithmetic

(c) $\vec{w_1} + \vec{w_4} = \frac{1}{2}(\vec{u} + \vec{v}) + \frac{1}{2}(\vec{u} - \vec{v}) = \vec{u}$.

Alternatively: shifting $\vec{w_4}$ to overlap with $\vec{w_2}$, we see that $\vec{w_1} + \vec{w_4}$ has the same start and end points as $\vec{u}$, so it equals $\vec{u}$.

(d) 0, because $w_3 = -\vec{w_1}$. (They have the same magnitude and point in opposite directions.)

(e) $\vec{w_3} + \vec{w_4} = -\vec{v}$, by an analog of either of the two explanations in (c).

1.4, A7 (ab).

(a) $\underbrace{\begin{bmatrix} 6 \\ 2 \\ 4 \\ -2 \end{bmatrix}}_{\vec{q}_1} = 2 \underbrace{\begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}}_{\vec{u}}$, so $\vec{q}_1$ and $\vec{u}$ are parallel.

(b) Suppose $\begin{bmatrix} -18 \\ -6 \\ 12 \\ 6 \end{bmatrix} = c \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$ for some scalar c.

Comparing the first coefficient, $c = -6$. But comparing the third coefficient, $c = 6$. These can't both be true, so the vectors are not parallel.

Note. We also know that $\vec{u}$ and $\vec{v}$ are parallel if and only if $\dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} = 1$ or -1. You can also solve the problem this way.