Quiz 10 - Math 544, Frank Thorne (thorne@math.sc.edu)

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Consider the matrix

$$M = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix}.$$

Compute M^{-1} and $\det(M)$.

$$\left[\begin{array}{ccc|ccc} 1 & -4 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{sub } 2R2 \text{ from } R1} \left[\begin{array}{ccc|ccc} 1 & -4 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

sub R1 from R3

$$\left[\begin{array}{ccc|ccc} 1 & -4 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 1 \end{array}\right]$$

Switch R2, R3

$$\left[\begin{array}{ccc|ccc} 1 & -4 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

Add 4·R2 to R1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 6 & 0 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

So $M^{-1} = \begin{bmatrix} -3 & 6 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$\det(M) = -\det\begin{bmatrix} 0 & 0 & 1 \\ 1 & -4 & 2 \\ 1 & -3 & 0 \end{bmatrix} = -1 \cdot \left(1 \cdot (-3) - 1 \cdot (-4)\right)$$

(switch R1, R3)

$$= -1 \cdot (-3 + 4) = -1 \cdot 1 = -1.$$