Quiz 3 - Math 544, Frank Thorne (thorne@math.sc.edu)

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Let P_2 be the vector space consisting of polynomials in t of degree at most 2. Determine whether each of these two sets are subspaces of P_2. Prove your claims.

(a)

$$S_1 = \{p(t) \in P_2 \ : \ p(t) = bt + ct^2 \text{ where b and c are real numbers with } b = -2c\}.$$

(b)

$$S_2 = \{p(t) \in P_2 \ : \ p(1) = 3\}.$$

(13 pts)

(a) It is. We need to check the three properties.

(1) The zero polynomial is $0t + 0t^2$ with $0 = -2 \cdot 0$ so it is in S_1.

(2) If $b_1 t + c_1 t^2$ and $b_2 t + c_2 t^2$ are two polynomials in S_1, then $b_1 = -2c_1$ and $b_2 = -2c_2$.

The sum of these polynomials is

$(b_1 + b_2)t + (c_1 + c_2)t^2$.

We have $b_1 + b_2 = -2c_1 - 2c_2 = -2(c_1 + c_2)$ so this is in S_1

(3) If $b_1 t + c_1 t^2$ is in S_1, and $a \in \mathbb{R}$, then

$a(b_1 t + c_1 t^2) = ab_1 t + ac_1 t^2$.

We have $ab_1 = a \cdot (-2c_1)$ (because $b_1 t + c_1 t^2 \in S_1$)

$= -2(ac_1)$ so $a \cdot (b_1 t + c_1 t^2) \in S_1$.

So S_1 satisfies the subspace axioms, and is a subspace.

(7 pts)

(b) If S_2 were a subspace, it would contain the zero polynomial $z(t)$. But $z(1) = 0 \neq 3$. So S_2 is not a subspace.

2.4. B6.

(a) $x=0,\ y=0,\ z=0.$

(b) Impossible, as $\begin{bmatrix}0\\0\\0\end{bmatrix}$ is a solution to any homogeneous system.

(c) $x=1,\ y=1,\ z=3.$

(d) $$\begin{aligned} x \quad + 2z &= 0 \\ y + z &= 0 \end{aligned}$$ works.

If $z=r$ then $y=-r$ and $x=-2r$ so the solution set is

$$\left\{\begin{bmatrix}-2r\\-r\\r\end{bmatrix} : r \in \mathbb{R}\right\} = \left\{\begin{bmatrix}2r\\r\\-r\end{bmatrix} : r \in \mathbb{R}\right\}$$
$$= \left\{r \cdot \begin{bmatrix}2\\1\\-1\end{bmatrix} : r \in \mathbb{R}\right\}.$$

(e) Impossible, as $\vec{0}$ is not in this set but it is a solution to any homogeneous system.

2.5 A 11.

$V = \mathbb{R}^3$, $Z = \left\{ \begin{bmatrix} 2a+5b \\ a \\ b \end{bmatrix} : a, b \in \mathbb{R} \right\}$.

Find a finite set Q with $Z = \mathrm{Span}(Q)$.

$Z = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \right\}$ is one possibility,

then $\mathrm{Span}(Z) = \left\{ r \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} : r, s \in \mathbb{R} \right\}$

which is the same as above.

There are other possibilities too.