**Quiz 4 - Math 544, Frank Thorne (thorne@math.sc.edu)**

**Monday, September 28, 2015**

Solve the system of linear equations

$$2x + y + 3z = 5$$

$$x + y + 2z = 3$$

$$-y + 3z = -5$$

via Gauss-Jordan elimination. Please use the following recipe:

(a) Write down an augmented matrix corresponding to the system above.

(b) Use row operations to bring it to row reduced echelon form (RREF).

(c) Write down the system of equations corresponding to your RREF matrix.

(d) Write down the solution set in as simple of a manner as possible.

The augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 1 & 1 & 2 & 3 \\ 0 & -1 & 3 & -5 \end{array}\right] \xrightarrow{\text{Switch R1 and R2}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 2 & 1 & 3 & 5 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

$$\text{Add } -2 \cdot R1 \text{ to } R2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

$$\text{Mul R2 by } -1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 3 & -5 \end{array}\right]$$

$$\text{Sub R2 from R1, Add R2 to R3} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 4 & -4 \end{array}\right]$$

$$\text{Mul R3} \times \tfrac{1}{4} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\text{Sub R3 from R1 and R2} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x = 3$

$y = 2$

$z = -1$

The solution set is the single vector $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.

2.5 B 11.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is one of many possibilities.

3.1 B7.

Is $r\vec{u} + s\vec{v} + t\vec{w} = \vec{b}$ ?

$$r\begin{bmatrix} 1 \\ 2 \end{bmatrix} + s\begin{bmatrix} 5 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} c \\ d \end{bmatrix} ?$$

$$r + 5s + 2t = c$$
$$2r + s - 3t = d$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 2 & c \\ 2 & 1 & -3 & d \end{array}\right]$$

Bonus. p. 267, 16.

Prove $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if and only if $ad-bc \neq 0$.

Suppose $ad-bc \neq 0$. Then $a$ and $c$ can't both be zero, since $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is row equivalent to $\begin{bmatrix} c & d \\ a & b \end{bmatrix}$, we may assume without loss of generality that $a \neq 0$.

Then, we have the following equivalences:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & b/a \\ c & d \end{bmatrix} \quad \text{(divide R1 by a)}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & b/a \\ 0 & d - cb/a \end{bmatrix} \quad \text{(subtract } c \cdot \text{R1 from R2)}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix} \quad \text{multiply R2 by } \left(d - \frac{cb}{a}\right)^{-1} = \frac{a}{ad-cb}, \text{ okay because } ad-cb \neq 0$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{sub } \frac{b}{a} \cdot \text{R2 from R1}$$

as desired.

Suppose now that $ad-bc=0$, we want to prove $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is not row equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(Note: This is the contrapositive of: If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is row equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $ad-bc \neq 0$. This is logically equivalent!)

(continued)

If both $a$ and $c$ are zero, then our matrix is

$\begin{bmatrix} 0 & b \\ 0 & d \end{bmatrix}$ which is not equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ as there will be no way to get a 1 ~~on the top~~ anywhere in the left column. So, as before, we may assume that $a \neq 0$.

We do the row reduction as before, but when we get to $\begin{bmatrix} 1 & b/a \\ 0 & d - cb/a \end{bmatrix}$, the bottom row is 0 and 0.

So this matrix is not row equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.