Quiz 2 - Math 544, Frank Thorne (thorne@math.sc.edu)

Wednesday, September 9, 2015

(1.5, B2). The equation

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \left\{ t \begin{bmatrix} 3 \\ 0 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \ : \ t \in \mathbb{R} \right\}$$

is a vector equation of a line in $\mathbb{R}^4$.

(a) Find three distinct points on the line.

(b) Find another vector equation of the same line. If possible (**hint: it is**), the new equation should use completely different numbers.

(a) We can plug in any t. So, say,

$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$ $(t=0)$, $\begin{bmatrix} 4 \\ 2 \\ -2 \\ 1 \end{bmatrix}$ $(t=1)$, $\begin{bmatrix} 31 \\ 2 \\ -20 \\ 19 \end{bmatrix}$ $(t=10)$.

(b) Start with any point on the line, say $\begin{bmatrix} 31 \\ 2 \\ -20 \\ 19 \end{bmatrix}$ and take any scalar multiple of $\begin{bmatrix} 3 \\ 0 \\ -2 \\ -2 \end{bmatrix}$, say twice it: $\begin{bmatrix} 6 \\ 0 \\ -4 \\ -4 \end{bmatrix}$.

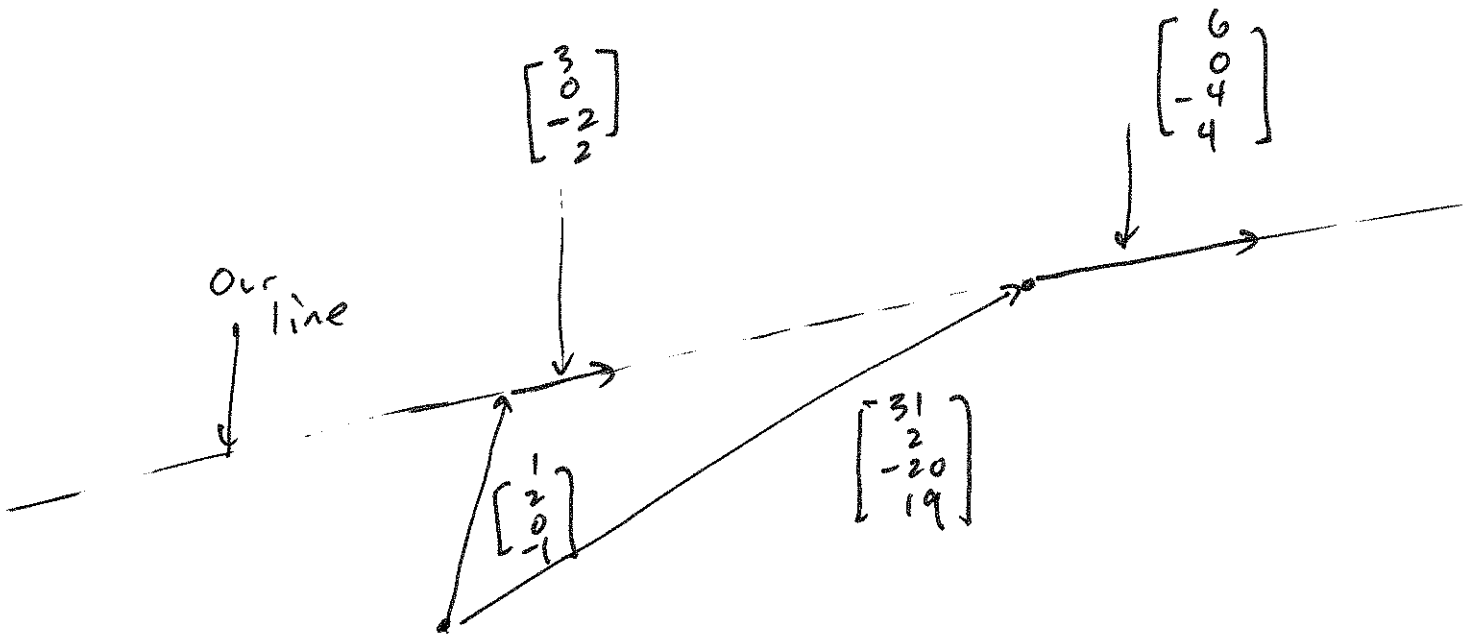
This point and direction again defines the same line,

$$\left\{ t \begin{bmatrix} 6 \\ 0 \\ -4 \\ 4 \end{bmatrix} + \begin{bmatrix} 31 \\ 2 \\ -20 \\ 19 \end{bmatrix} \ : \ t \in \mathbb{R} \right\}.$$

Picture on next page

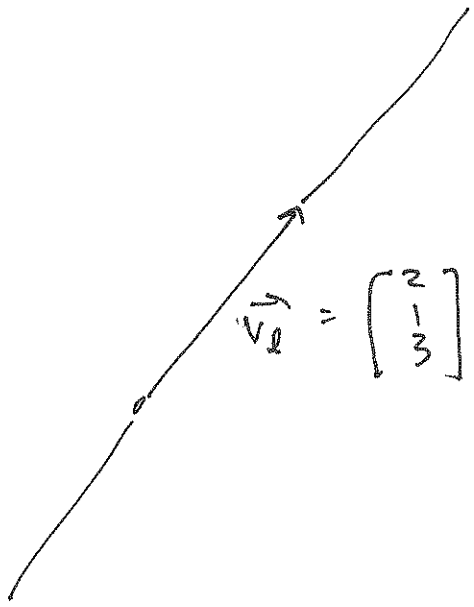
Note: You can't avoid reusing the 0 and the 2!

Extra credit for pointing this out.


$\begin{bmatrix} 3 \\ 0 \\ -2 \\ 2 \end{bmatrix}$
$\begin{bmatrix} 6 \\ 0 \\ -4 \\ 4 \end{bmatrix}$
Our line
$\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$
$\begin{bmatrix} -31 \\ 2 \\ -20 \\ 19 \end{bmatrix}$

1.5.

A1.



$$L = \left\{ t \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} : t \in \mathbb{R} \right\}.$$

(This is (a) and (b). See alternatively the back of the book.)

$\vec{u_1}$, $\vec{u_2}$ ~~$\vec{u_4}$~~ one: take $t = 4, -1$ ~~5~~ respectively.

$\vec{u_3}$ is not. If $\begin{bmatrix} -6 \\ 2 \\ 9 \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

then $6 = 2t$ so $t = 3$

and $2 = 1 \cdot t$ so $t = 2$.

This is impossible.

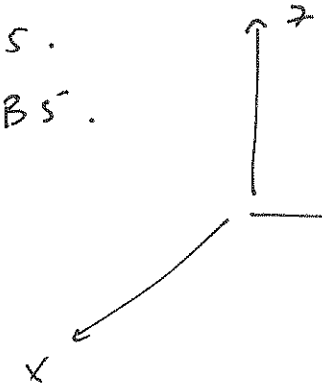
$\vec{u_4}$ is not. If $\begin{bmatrix} -10 \\ 5 \\ -15 \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

then $t = -5$ and $t = 5$, impossible.

Note. For $\vec{u_3}$ and $\vec{u_4}$, I also accepted the briefer ~~so~~ explanation "because no value of t works".

1.5.

B5.



This is the plane $y=0$.

So any vector of the form $\begin{bmatrix} x \\ 0 \\ z \end{bmatrix}$.

We can write this as

$$\left\{ \begin{bmatrix} r \\ 0 \\ s \end{bmatrix} : r, s \in \mathbb{R} \right\}$$

$$= \left\{ r \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} : r, s \in \mathbb{R} \right\}.$$

Alternatively:

$$\left\{ r \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} : r, s \in \mathbb{R} \right\}.$$