Homework 0 - Math 574, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, January 14

This is a preliminary homework, which will let me know where you are starting from and which will give you an idea for what we will study in this course. **Please do the best you can, do not use any books or notes, and don't take longer than three hours total.** If you appear to have made a reasonable effort, you will get full credit for this assignment.

If you have difficulties (or no idea) with any of the problems, then please plan on putting in hard work, and I will do my best to help you master this material!

(a) Consider the sets $S = \{a, b, x\}$ and $T = \{c, x\}$. List all the subsets of S and of T. What are the union and intersection of S and T? Describe a function (your choice) from S to T.

(b) Socrates is a man. All men are mortal. Does it follow that Socrates is mortal?

(c) Aristotle is mortal. All men are mortal. Does it follow that Aristotle is a man?

(d) Prove the following: For every integer n, there exists an integer m which is bigger than n.

(e) Prove that $1 + 2 + 3 + 4 + \cdots = \frac{n(n+1)}{2}$.

(f) Prove that the sum of two odd numbers is always even.

(g) Prove that there is no positive rational number (i.e., fraction) which is smaller than all other positive rational numbers.

(h) South Carolina license tags consist of three letters and three numbers. How many different license tags can be created?

(i) Suppose that no letter can be repeated. How many different license tags can be created now?

(j) You are walking in a city which is laid out in a grid. You want to walk four blocks north and four blocks east. At every step you can choose whether to walk north or east. For example, you could walk four blocks east, then four blocks north; or one block east, two blocks north, one block east, two blocks north, two blocks east. Any direction is fine.

How many different ways are there to reach your destination?