Homework 4 - Math 574, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, February 17 at 5:00.

The discussions on writing proofs and common mistakes in Chapter 4.1 of your book are extremely useful. I strongly recommend that you read them carefully, especially if you are unfamiliar with writing proofs.

Core:

4.1: 5, 11, 14, 18 (but only for $1 \leq n \leq 5$), 24, 31, 44, 45, 53.

4.2: 14, 17, 18, 28.

4.6: 3, 5, 10, 12, 15, 19, 21.

Determine whether each of the following statements is true or false, and prove or disprove each of them. (Feel free to apply the division with remainder theorem.)

1. If n is any odd integer, then n^2 is equal to $16a + b$ for some integers a and b.
2. If $n = 3k + 1$ for some integer k, then n^2 is equal to $3m + 1$ for some integer m.
3. If $n = 3k + 1$ for some integer k, then n^2 is equal to $9m + 1$ for some integer m.
4. If $n = 4k + 1$ for some integer k, then n^2 is equal to $8m + 1$ for some integer m.
5. If $n = 4k + 1$ for some integer k, then n^2 is equal to $16m + 1$ for some integer m.

Additional:

4.1: 12, 15, 25, 33, 43, 46.

4.2: 8, 12, 16.

4.6: 4, 7, 11, 24.

Bonus:

1. (3 points; partial credit for some special cases) Determine, with proof, for which integers m the following statement is true:

   For each integer n that satisfies $n = am + 1$ for some integer a, we have $n^2 = bm^2 + 1$ for some integer m.

2. (2 points) Prove:

   Let $P(x) = a_k x^k + a_{k-1} x^{k-1} + \cdots + a_1 x$ be a polynomial in n without a constant term. Then, $P(n)$ is even for all integers n if and only if the sum $a_1 + a_2 + \cdots + a_k$ is even.

3. (2 points) 4.2, 32.