Homework 5 - Math 574, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, February 24 at 5:00.

There are shortcuts on this week's homework. If this starts to get tedious and repetitive, look for them!

For epsilon-delta problems, if you graph your function you may assume that a function $f(x)$ is increasing on an interval $[a, b]$ if it is clearly visible from your graph. For example, $f(x) = x^2$ is increasing on $[1, 2]$.

1. Prove that $\sqrt{3}$ is irrational.
2. Prove that $\sqrt{5}$ is irrational.
3. Prove that $\sqrt{6}$ is irrational.
4. Prove that $\sqrt{2} + 2$ is irrational.
5. Prove that $48\sqrt{5} + 2$ is irrational.
6. Prove that $\sqrt[3]{2}$ is irrational.
7. Prove that $\sqrt[3]{3}$ is irrational.
8. Prove that $\lim_{x \to 2} 0 = 0$.
9. Prove that $\lim_{x \to 5} 3x = 15$.
10. Prove that $\lim_{x \to 3} 4x - 1 = 11$.
11. Prove that $\lim_{x \to 2} -2x - 9 = -13$.
12. Prove that $\lim_{x \to 5} 0 \neq 1$.
13. Prove that $\lim_{x \to 4} 2x + 1 \neq 10$.
14. Prove that $\lim_{x \to 2} x^2 \neq 3$.
15. Prove that $\lim_{x \to \frac{\pi}{4}} \sin(x) \neq 1$.
16. A few additional problems will be added later (there will be a few more problems from Chapter 4 of Epp's book.)

Bonus (harder $\epsilon - \delta$ proofs)

1. (2 points) Prove that $\lim_{x \to 3} x^4 = 81$.
2. (2 points) Prove that $\lim_{x \to 0} \sin(1/x)$ does not exist.
3. (2 points) Prove that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$. (Do not use L'Hopital's Rule.)