580, HW 2.

11. (a) verify $2^5 \cdot 9^2 = 2592$.

$$2^5 \cdot 9^2 = 32 \times 81$$

$$\begin{array}{r} 32 \\ \underline{81} \\ 32 \\ \underline{256} \\ 2592 \end{array}.$$

(b) Is $2^5 a^b = 25ab$ possible for other a and b?

<u>what we know:</u>

$2^5 \cdot a^b = 32a^b$ has to be between 2500 and 2599,

So a^b is 79, 80, 81.

79 is prime and so not of this form.

80 is not ~~a power~~ of this form because $80 = 16 \cdot 5 = 2^4 \cdot 5$.

It can't be a kth power for $k > 1$, because then 5^k would have to appear.

This leaves only the solution above.

p. 26, #1.

$x + y = 2$: x can be anything, then $y = 2 - x$.

$3x - 4y = 5$. By inspection, $x = 3$, $y = 1$ is a solution.

Then all solutions are given by

$$\begin{aligned} x &= 3 + 4t \\ y &= 1 + 3t \end{aligned} \quad \text{for } t \in \mathbb{Z}.$$

$15x + 16y = 17$. Need to find one solution first.

Note that $15 \cdot (-1) + 16 \cdot (1) = 1$.

So $15 \cdot (-17) + 16 \cdot (17) = 17$.

So all solutions are

$$\begin{aligned} x &= -17 + 16t \\ y &= 17 - 15t \end{aligned} \quad \text{for } t \in \mathbb{Z}.$$

p. 26 #9.

Let a = amount that merchant #1 has

b = " " " #2 has

c = " " " #3 has

p = amount in the purse

Then

$$a + p = 2(b+c) \Rightarrow a - 2b - 2c + p = 0. \quad (\#1)$$

$$b + p = 3(a+c) \Rightarrow -3a + b - 3c + p = 0 \quad (\#2)$$

$$c + p = 5(a+b) \Rightarrow -5a - 5b + c + p = 0. \quad (\#3)$$

Do Gaussian elimination.

Subtract #3 from #2: $2a + 6b - 4c = 0$ (#4)

#2 from #1: $4a - 3b + c = 0$. (#5)

Add 4 times #5 to #4: $18a - 6b = 0$

$3a - b = 0$ so $\boxed{b = 3a.}$

Have $2a + 6b - 4c = 0$

$2a + 18a - 4c = 0$

$20a - 4c = 0$

$5a - c = 0$ so $\boxed{c = 5a}$

Given $a - 2b - 2c + p = 0$

$a - 6a - 10a + p = 0$ $\boxed{p = 15a.}$

The solution is $a =$ any positive integer

$b = 3a$

$c = 5a$

purse $= 15a$.

Note that this wasn't really a number theory question.

2. mod 7 :

x	0	1	2	3	4	5	6
x^2	0	1	4	2	2	4	1

So the list is 0, 1, 2, 4 (mod 7).

The bare answers:

(mod 5) : 0, 1, 4

(mod 7) : 0, 1, 2, 4

(mod 11) : 0, 1, 3, 4, 5, 9

(mod 13) : 0, 1, 3, 4, 9, 10, 12.

Some observations.

- If p is prime, ($p \neq 2$) the list (mod p) always contains $\frac{p+1}{2}$ numbers

- If $p \equiv 1 \pmod 4$, the list is always symmetric! If $x \pmod p$ is in the list, then $-x \pmod p$ is too.

- If $p \equiv 3 \pmod 4$ the list is antisymmetric: If x is on the list, $-x$ isn't, and vice versa.

Gauss's Big Secret to come!

Ch. 4, #5.

Want $m \mid 1776 - 1066 = 710$

So m has to be a divisor of 710.

Since $710 = 2 \cdot 5 \cdot 71$ with 71 prime,

the divisors are 1, 2, 5, 71, 10, 142, 355, 710.

#18. A four-digit palindrome is of the form

$1000a + 100b + 10b + a$ for some a, b

$= 1001a + 110b$

$= 11(91a + 10b)$.

A six digit palindrome is of the form

$100000a + 10000b + 1000c + 100c + 10b + a$

$= 100001a + 10010b + 1100c$

$= (110000 - 9999)a + (11000 - 990)b + 1100c$

$= 11[(10000 - 909)a + (1000 - 90)b + 100c]$.

The same is true!

Ch 5.

12. A multiple of 7 that leaves the remainder 1 when divided by 2, 3, 4, 5, or 6.

Note that 60 is a multiple of 2, 3, 4, 5, 6

So enough to solve

$$7x \equiv 1 \pmod{60}.$$

Or, altenatively,

$$7x = 60y + 1$$

So solve $60y \equiv -1 \pmod 7$ instead

$$4y \equiv 6 \pmod 7$$

$$2y \equiv 3 \pmod 7$$

$$2y \equiv 10 \pmod 7$$

$$y \equiv 5 \pmod 7$$

So $y = 30$ works

$$7 \cdot 43 = 5 \cdot 60 + 1.$$